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Readiness or Resources: Which Comes First?

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CENTER·FOR·NAVAL·ANALYSES

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INTRODUCTION

This ~~paper~~ report concerns a statistical problem in estimating relationships between resources and readiness. In policy language, the question is which term, resources or readiness, is the discretionary, or causal variable, and which is the determined, or effect variable. In statistical terms, the question is which variable to put on the left-hand side of a statistical regression relationship. Although this issue could arise in studying any area of the budget, we will discuss it in the context of the Naval shore establishment.

RELATING RESOURCES TO READINESS FOR NAVAL SHORE BASES

To help manage the shore establishment, the Navy has been taking steps to relate resources to readiness for its bases. Such relationships can be useful in determining the cost of bringing deficient bases up to a desired readiness level, or allocating a given BOS budget across bases in order to achieve a uniform state of readiness.

This ~~paper~~ report is concerned with the analytical methodology for relating readiness to spending. We will assume the Navy has managed to obtain reasonable measures of readiness.

The question, therefore, is how estimating the relationship between the readiness (R) and BOS cost (C) of a base (in either a cross-section or a time-series analysis.)

MODEL

To obtain causal relationships that can be applied to policy questions; we first need a model that describes our best understanding of the underlying mechanisms. We will assume that the Navy's management of the shore establishment can be described in the aggregate by the following two-phase model. The Navy is given a total budget based on broad objectives ("We need a 600-ship Navy"), and the CNO divides up the budget based on requests for support by the sponsors of the shore establishment and the other activities that comprise the Navy. How much is given to each base (C) is based on several types of variables: the installation's current readiness (R), its physical and personnel characteristics (building area, number of military people, etc.)

represented by a size variable (S), a host of other factors that enter the allocation of the Navy budget (Z), and finally an error term (u):

$$C = a_0 + a_1 R + a_2 S + a_3 Z + u \quad . \tag{1}$$

In the "spending" phase of the management process, each base spends its budget in such a way as to maximize readiness. The readiness (R) a base actually achieves depends not only on its budget, but also on its characteristics (S), a variety of other factors (W), and finally an error term (v):

$$R = b_0 + b_1 C + b_2 S + b_3 W + v \quad . \tag{2}$$

This is a simultaneous equation model with no specification as to dynamics—how the "allocation" and "spending" phases occur in time. In reality, what the Pentagon gives a base to spend at the start of a given fiscal year depends on how much readiness the base has already achieved as a result of prior years' allocations. A base, moreover, can receive budgetary adjustments during the year to handle unforseen problems that may arise. (It is possible that the problem discussed in this paper would not arise in a time-dependent analysis with small enough time intervals. Simultaneous models are nevertheless necessary because such detailed time-series data are typically unavailable.)

Estimating the full two-step model (equations 1 and 2) as it stands is out of the question, at least at present. It would be a major project to define and measure the "other factors" (Z) that strongly determine how much BOS funds the Navy gives to each base.

ESTIMATING THE "BASE SPENDING" EQUATION

It is not out of the question, however, to estimate equation 2, the "base spending" equation. Data on cost (C), readiness (R), and various size variables (S) are available from the budget, the new BaseRep system, and other sources. We would need to omit the "other factors" (W) and see if the explanatory power is still satisfactory. "Other factors" are intuitively less important in determining the readiness achieved with a given budget (equation 2) than in determining the budget itself (equation 1).

There is, however, the question of *how* to estimate the base spending equation, equation 2 (without the W). We are not concerned, here, with questions like whether to use a linear or log-linear specification, but rather the more fundamental question of which variable, C or R, to treat as endogenous—to put on the left-hand side. The fact that equation 2 has R on the left-hand side does not imply that the equation must be estimated this way. One could instead estimate:

$$C = d_0 + d_1R + d_2S + e \ . \tag{3}$$

The choice of whether to estimate equation 2 (without the W) or 3 is called the "normalization" problem in the statistics literature. Roger Klein shows that the choice should reflect the criterion for ordinary least squares: right-hand variables should be uncorrelated with the error term. The full model implies, of course, that neither C nor R is *rigorously* uncorrelated with v; v affects R through equation 2, and thereby C through equation 1. The question is which variable, C or R, is *less* correlated with v.

One way to decide this is to take the full model and solve the equations for C and R; then calculate the correlation coefficients $\rho^2(v, C)$ and $\rho^2(v, R)$ in terms of the unknown coefficients of the model; and finally, use our intuition about these coefficients to judge which ρ^2 term is larger. If $\rho^2(v, C) < \rho^2(v, R)$ then C is more exogenous (less dependent on v) and should be put on the right-hand side. We would thus estimate equation 2 (without the W). In the opposite case, we would estimate equation 3.

We can do the same thing less formally. Suppose we assume, for example, that a_1 in equation 1 is small—that the principal determinant of a base's budget is its size, with readiness being an adjustment to this basic relationship. In this case, equation 2 would be the one that should be estimated. Look at the full model and assume that v changes. This affects R by equation 2 and thereby C by equation 1. The last link is weak, however, given our assumption that a_1 is small. The fact that v has little effect on C means that C is more exogenous than R, and should thus be on the right-hand side. The question of just how small a_1 must be is derived in the last section.

Alternatively, if we assume a_2 is small—that bases with low readiness get much more funding and size is now the adjustment—then v and C are closely linked and equation 3 is the one that should be estimated.

Note that choosing an underlying model is the *only* way to select which equation to estimate. Looking only at goodness of fit is inappropriate, and sometimes misleading. For example, taking a_1 small, which leads to putting R on the left-hand side, also suggests that putting C on the left-hand side would yield "better" statistical results. If the effect of S on C is the dominant relationship, one would obtain a higher R^2 from putting C, rather than R on the left in equation 2. This would also avoid multicollinearity problems between C and S. These are not, however, the appropriate criteria for choosing between models with different variables. Theory is the correct criterion.

"INVERTED PRODUCTION FUNCTION" MODEL

There is an alternative approach to relating BOS cost to readiness. The technique involves inverting the base's production function.

Regard the base characteristics (S) as outputs of the BOS resources. The square feet of floor space, for example, is one measure of output maintenance resources and utilities the base employs, holding readiness constant. More generally, the base production function relates output to the factors of capital and labor employed (readiness held constant):

$$S = f(K,L) = MK^a L^b \; . \tag{4}$$

Next is the cost of capital and labor at given prices ("r" for implicit rent, "w" for wage rate):

$$C = rK + wL \; . \tag{5}$$

The next step is to minimize cost subject to a constraint on output, S. The Lagrangian method gives the solution[1]:

$$C = HS^{\left(\frac{1}{a+b}\right)} r^{\left(\frac{a}{a+b}\right)} w^{\left(\frac{b}{a+b}\right)} \; , \tag{6}$$

where

$$H = (a+b)\left[\frac{1}{M}\left(\frac{1}{a}\right)^a\left(\frac{1}{b}\right)^b\right]^{1/(a+b)} \; .$$

1. Hans Brems, *Quantitative Economic Theory*, Wiley 1968. (HB-74-M3-B735), p. 78.

BOS funding level is thus a function of the workload and the coefficients of the production function (still holding readiness constant).

Since we might use a variety of workload measures, and since the coefficients of the production function are unknown, we end up with a cost function to be estimated by regression analysis:

$$C = d_0 + d_1S_1 + d_2S_2 + \dots + e \ . \tag{7}$$

The final step is to recognize that we would obtain different cost functions for different levels of readiness. A million square feet can be maintained in good shape for $\$X$ million, and in better shape for $\$(X+\Delta)$ million. We can reflect this dependence by expanding the cost function to include readiness as an explicit variable (i.e., readiness as simply another output):

$$C = d_0 + d_1S_1 + d_2S_2 + \dots + d_RR + e \ . \tag{8}$$

Even under this alternate specification, there is still the basic issue of which variable belongs on the left hand side and which belongs on the right-hand side.[1]

SPECIFICATION OF THE "BASE SPENDING" EQUATION: RIGOROUS TREATMENT

In this section we compare the two versions of the "base spending" equation to be estimated in terms of formal statistical properties:

$$R = b_o + b_1C + b_2S + v \tag{9}$$

$$C = d_0 + d_1R + d_2S + e \ . \tag{10}$$

Note that equations 9 and 10 are exactly the same equation except that the variables C, R have been interchanged. The parameters in equations 9 and 10 are obviously closely related. For example, $b_1 = 1/d_1$.

1. There is, of course, a mathematical duality between maximizing output subject to a cost constraint and minimizing cost subject to an output constraint. The duality does not carry over, however, to statistics: because the error term is not related equally to the left-hand and right-hand variables, it does matter for normalization whether cost or output is put on the right-hand-side.

The question for analysis is which version is preferred on statistical grounds.

To compare equations 9 and 10, they must be combined with the other equation in the system, the reaction function. We will analyze equation 9 along with equation 1 from the previous section.

The system to be analyzed is thus:

$$C = a_o + a_1R + a_2S + a_3Z + u \tag{1}$$

$$R = b_o + b_1C + b_2S + v \ . \tag{9}$$

The question of whether to estimate equation 9 or 10 comes down to the question of whether R or C is less correlated with v and thus will cause the least damage on the right hand side. More formally, Klein[1] shows that (aside from a term that vanishes as $\sigma_v^2 \rightarrow 0$) the mean square errors (MSE) of the parameter estimates of b_0, b_1, b_2 will be minimized by putting on the right-hand side the variable with the lesser correlation with the right-hand side error term, v. Note that the MSE criterion depends on the initial normalization, i.e., the analyst must decide whether he is interested in the MSE of the parameters of the production function or the cost function. The optimal normalization seems likely to be the same regardless of the initial normalization, but this is not guaranteed. We have chosen the initial normalization with R as the dependent variable, since this is the form with the policy relevance (the policy issue is the effect of C on R).

We now turn to determining which variable R or C has the lower correlation with v. We start by rewriting the system in a more compact notation:

$$f + Ay + Bx + e = 0 \ ,$$

1. Roger W. Klein, "K-Class Estimators: The Optimum Normalization for Finite Samples," *Journal of the American Statistical Association*, June 1973, Vol. 68, No. 342, pp. 445-451.

where

$$A = \begin{bmatrix} -1 & a_1 \\ b_1 & -1 \end{bmatrix}$$

$$y = \begin{bmatrix} C \\ R \end{bmatrix}$$

$$x = \begin{bmatrix} S \\ Z \end{bmatrix}$$

$$B = \begin{bmatrix} a_2 & a_3 \\ b_2 & 0 \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$f = \begin{bmatrix} a_c \\ b_c \end{bmatrix}.$$

To examine correlations with the residuals, it is necessary to derive the reduced form:

$$y = (-A)^{-1}(f + Bx + \varepsilon)$$

The covariance matrix between y and ε can be evaluated as $E(y\varepsilon')$ because $E(\varepsilon) = 0$.

$$\begin{bmatrix} \sigma(C,u) & \sigma(C,v) \\ \sigma(R,u) & \sigma(R,v) \end{bmatrix} = E(y\varepsilon')$$

$$= E[(-A)^{-1}(f + Bx + \varepsilon)\varepsilon']$$

$$= (-A)^{-1}E(\varepsilon\varepsilon') = (-A)^{-1}\begin{bmatrix} \sigma^2(v) & 0 \\ 0 & \sigma^2(v) \end{bmatrix},$$

where we have assumed that $\sigma(u,v) = 0$.

Define $G = (-A)^{-1}$ with typical element g_{ij}:

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}.$$

With this notation

$$E(y\varepsilon') = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \sigma^2(u) & 0 \\ 0 & \sigma^2(v) \end{bmatrix}.$$

The second column of $E(y\varepsilon')$ is $\begin{bmatrix} \sigma(C,v) \\ \sigma(R,v) \end{bmatrix} = \begin{bmatrix} g_{12}\sigma^2(v) \\ g_{22}\sigma^2(v) \end{bmatrix}.$

R will be preferred as the left-hand side variable provided that $\rho^2(R,v) > \rho^2(C,v)$. Klein shows that an equivalent condition is that:

$$1 + 2b_1 \, cov\,(m_2^*, v^*) \geq 0$$

where $v^* = \dfrac{v}{\sigma(v)}$ and $m_2^* = m_2/\sigma(v)$ and m_2 is the residual in the reduced form equation for C.

Thus, the condition is rewritten

$$1 + \frac{2b_1}{\sigma^2(v)} \, cov\,(m_2, v) \geq 0$$

$$= 1 + \frac{2b_1}{\sigma^2(v)} \, cov\,(C, v) \geq 0$$

$$= 1 + \frac{2b_1}{\sigma^2(v)} \, g_{12}\sigma^2(v) \geq 0$$

$$= 1 + 2b_1 g_{12} \geq 0 \quad .$$

At this point (to evaluate g_{12}) we need to evaluate the matrix $G = (-A)^{-1}$.

$$A = \begin{bmatrix} -1 & a_1 \\ b_1 & -1 \end{bmatrix}$$

$$-A = \begin{bmatrix} 1 & -a_1 \\ -b_1 & 1 \end{bmatrix}$$

$$G = \left[\frac{\begin{matrix} 1 & a_1 \\ b_1 & 1 \end{matrix}}{1 - a_1 b_1} \right] .$$

so that $g_{12} = (1 - a_1 b_1)^{-1} a_1$.

The condition for R to be the preferred left-hand variable is now:

$$1 + 2b_1 g_{12} \geq 0$$

$$1 + 2b_1 a_1 (1 - a_1 b_1)^{-1} \geq 0$$

or

$$\frac{1 + a_1 b_1}{1 - a_1 b_1} \geq 0 \ .$$

This condition is satisfied so long as $|a_1 b_1| < 1$. If a_1 and b_1 are positive, the requirement (for R to be the left-hand variable) is that a_1 be small in the sense that $a_1 \leq \frac{1}{b_1}$

Graphical Analysis

The conditions we have derived can be illustrated graphically. The solid line is the production function for readiness, the dotted line the reaction function.

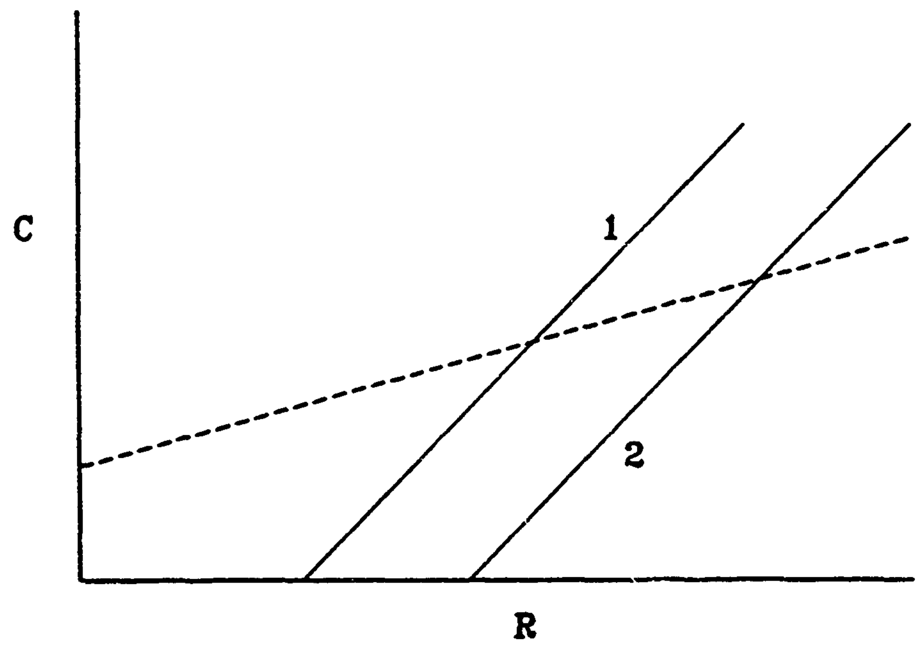

Fig. 1: Graphical Analysis of the Normalization Condition

Suppose 1 corresponds to $v = 0$. When v takes a positive value 1 shifts to 2. Because of this increase in v, both R and C increase, i.e., they both have a positive correlation with the error term. The correlation of R and v will be large relative to that between C and v if the dotted line is flat. If the dotted line is horizontal, C does not vary at all, i.e., it is uncorrelated with v.

What the figure indicates is that a low value of a_1 (the slope of the dotted line), leads to R being the more endogenous variable and thus the preferred left-hand variable.

Summary

The decision of which equation to estimate, (2) or (3)—the "normalization" question)—depends on whether C or R is more correlated with the error term, *not on the value of* R^2.

Theoretically, either normalization is possible.

In practice, we suspect that b_1 is small: a base's readiness is a function of many years past spending and extra funds this year will not make a big change. If b_1 is small, then $1/b_1$ is large. Further, a_1 is likely to be small: a base's budget is not heavily dependent on its readiness. Thus a_1 is likely to be less than $1/b_1$ so that R is the appropriate left-hand-side variable.

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PP 416
Jondrow, James M., and Levy, Robert A. *The Displacement of Local Spending for Pollution Control by Federal Construction Grants*, 6 pp., Jun 1984 (Reprinted from *American Economic Review*, May 1984)

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1. CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the Management Information Office, Center for Naval Analyses, 4401 Ford Avenue, Alexandria, Virginia 22302-0268. An index of selected publications is also available on request. The index includes a listing of professional papers, with abstracts, issued from 1969 to December 1983).
2. Listings for Professional Papers issued prior to PP 407 can be found in *Index of Selected Publications (through December 1983)*, March 1984.

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Berg, Robert, Dennis, Richard, and Jondrow, James. *Price Analysis and the Effects of Competition*, 23 pp., Sep 1985. (Presented at the Association for Public Policy Analysis and Management–The Annual Research Conference, Shoreham Hotel, Washington, D.C., 25 October 1985)

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Ernst, Maurice E. (Hudson Institute). *Implications of Global Economic Forces for Political Instability in Selected Third World Countries*, 37 pp., Jul 1985